Chöông I :

VAT LY SONG

I. PHÖÔNG TRÌNH SONG ÑIEN TÖ:

1. Cac phöông trình Maxwell:

Tröông ñien tö trong chan khong vao thôi ñiem t nao ño ñöôc xac ñònh bôi vectô cöông ño ñien tröông $\vec{E}(\vec{r},t)$ va vectô cam öng tö $\vec{B}(\vec{r},t)$ vôi $\vec{r}$ la vectô vò trí tai ñiem ñang xet.

Löc tac dung len ñien tích thö Q chuyen ñong vôi van toc $\vec{v}$ ñöôc bieu dien thong qua $\vec{E}$ va $\vec{B}$ nhö sau:

$$\vec{F} = Q\vec{E} + Q(\vec{v} \wedge \vec{B}) \qquad \text{(löc Lorentz)}$$

Nguon cua tröông ñien tö la cac ñien tích va dong ñien , ñe ñac tröng cho cac ñai löông ño ngöôi ta dung mat ño ñien tích ρ va vectô mat ño dong ñien $\vec{j}$.

Cac phöông trình Maxwell bieu dien moi lien he giöa sö bien thien cua tröông ñien tö $(\vec{E},\vec{B})$ vôi cac nguon cua no (ñien tích, dong ñien) :

- Phöông trình Maxwell-Flux (M-Φ) : div$\vec{B}$ = 0 (bao toan tö thong)
- Phöông trình Maxwell-Faraday (M-F) : $rot\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (cam öng ñien tö)
- Phöông trình Maxwell-Gauss (M-G) : div$\vec{E}$ = $\frac{\rho}{\varepsilon_0}$ (ñònh ly Gauss)
- Phöông trình Maxwell-Ampere : $rot\vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ (ñònh ly Ampere)

$\vec{j}$: dong ñien dan

$\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$: dong ñien dòch

$\varepsilon_0 = \frac{1}{36\pi}.10^{-9}$ (F/m) : hang so ñien

$\mu_0 = 4\pi.10^{-7}$ H/m : hang so tö

2. Cac phöông trình lan truyen song:

Thöc hien phep toan rotor ñoi vôi phöông trình M-F :

$$rot(rot\vec{E}) = rot(-\frac{\partial \vec{B}}{\partial t}) = -\frac{\partial}{\partial t}(rot\vec{B})$$

Mat khac :

$$rot(rot\vec{A}) = grad(div\vec{A}) - \Delta\vec{A}$$

Tö phöông trình M-A, lay ñao ham hai ve theo thôi gian :

$$-\frac{\partial}{\partial t}(rot\vec{B}) = -\mu_0\frac{\partial\vec{j}}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Rightarrow \Delta\vec{E} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = grad(\frac{\rho}{\varepsilon_0}) + \mu_0\frac{\partial\vec{j}}{\partial t} \qquad \text{(a)}$$

Mot cach töông tö, tac dung toan tö rotor vao hai ve phöông trình M-A ta lai co:

$$rot(rot\vec{B}) = \mu_0 rot\vec{j} + \mu_0\varepsilon_0 rot(\frac{\partial\vec{E}}{\partial t}) = \mu_0 rot\vec{j} + \mu_0\varepsilon_0\frac{\partial}{\partial t}(rot\vec{E})$$

Lay ñao ham theo thôi gian phöông trình M-F :

$$\mu_0\varepsilon_0\frac{\partial}{\partial t}(rot\vec{E}) = -\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\Rightarrow grad(div\vec{B}) - \Delta\vec{B} = \mu_0 rot\vec{j} - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\Leftrightarrow \Delta\vec{B} - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} = \mu_0 rot\vec{j} \qquad \text{(b)}$$

(a) va (b) la cac phöông trình lan truyen cua tröông

3. Tröông hôp khong co nguon : ($\rho = 0, \vec{j} = 0$)

Cac phöông trình lan truyen cua ñien tröông $\vec{E}$ va tö tröông $\vec{B}$ luc ño co dang cua phöông trình D'Alembert:

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \;;\quad \Delta\vec{B} - \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} = 0 \qquad \text{(c)}$$

Vôi $c^2 = 1/\varepsilon_0\mu_0$:van toc truyen trong chan khong.

Toan tö D'Alemert: $\square = \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t}$

- ⇨ $\square\vec{E} = 0$; $\square\vec{B} = 0$
- ⇨ Ñoi vôi moi thanh phan cua tröông $\vec{a}$ co the bieu dien döôi dang $\square\vec{a} = 0$

4. Cac the cua tröông :

Ta co : div(rot $\vec{A}$) = 0

Tö phöông trình M-Φ : $div\vec{B} = 0$

=> ton tai mot tröông vectô $\vec{A}$ sao cho : $\vec{B} = rot(\vec{A})$

Mat khac, $rot(gradV) = 0$

Tö phöông trình M-F :

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(rot\vec{A}) = -rot(\frac{\partial \vec{A}}{\partial t})$$

$$\Rightarrow rot(\vec{E} + \frac{\partial \vec{A}}{\partial t}) = 0$$

=> tröông vectô $\vec{E} + \frac{\partial \vec{A}}{\partial t}$ la tröông xoay, vay ton tai mot tröông vo hööng V sao cho :

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -gradV$$

Tom lai ,tröông ñien tö $(\vec{E}, \vec{B})$ co mot cap the $(\vec{A}, V)$ lien he vôi chung qua bieu thöc :

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - gradV \text{ ;} \qquad \vec{B} = rot\vec{A}$$

Nhan xet rang, neu $\vec{A}$ la vectô the cua tröông ñien tö thì :

$\vec{A}$' = $\vec{A}$ + gradf cung la vectô the (f la mot ham so bat ky)

Va neu V la the vo hööng cua tröông thì

V' = V - $\frac{\partial f}{\partial t}$ cung la the vo hööng.

Trong so nhöng cap the cua mot tröông ñien tö xac ñònh, ton tai mot cap the thoa ñieu kien chuan Lorentz :

$$div\vec{A} + \mu_0 \varepsilon_0 \frac{\partial V}{\partial t} = 0$$

Khi ño :

$$\frac{\rho}{\varepsilon_0} = div\vec{E} = -div(gradV + \frac{\partial A}{\partial t}) = -\Delta V - \frac{\partial}{\partial t}(div\vec{A})$$

$$\Rightarrow \Delta V - \varepsilon_0 \mu_0 \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon_0} \qquad \text{(c)}$$

Va

$$rot\vec{B} = rot(rot\vec{A}) = \mu_0(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}) = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{j} - \mu_0 \varepsilon_0 grad(\frac{\partial V}{\partial t}) - \varepsilon_0 \mu_0 \frac{\partial^2 A}{\partial t^2}$$

$$grad(div\vec{A}) - \Delta\vec{A} = \mu_0 \vec{j} + grad(div\vec{A}) - \varepsilon_0 \mu_0 \frac{\partial^2 A}{\partial t^2}$$

$$\Rightarrow \Delta\vec{A} - \varepsilon_0 \mu_0 \frac{\partial^2 A}{\partial t^2} = -\mu_0 \vec{j} \qquad \text{(d)}$$

(c) va (d) la cac phöông trình lan truyen cua cac the.

II. SONG ÑIEN TÖ PHANG CHAY ÑIEU HOA LIEN TIEP (OPPH – Ondes Planes Progressives Harmoniques) :

1. Mô ñau:

- Mat song : la tap hôp cac vò trí ma ño lôn cua tröông khong ñoi vao thôi ñiem xac ñònh.

- Song phang (OP) la song co mat song la mot ho cac m at phang vuong goc vôi phöông truyen song xac ñònh $\vec{u}(|\vec{u}| = 1)$

- Song phang lien tiep (OPP) la song phang truyen theo phöông va chieu xac ñònh, ham song co dang:

$$a(M,t) = f(\vec{u}\vec{r} - ct)$$

Nghiem cua phöông trình D'Alembert la to hôp cac song phang lien tiep theo mot phöông $\vec{u}$ nao ño .

- Song phang ñieu hoa lien tiep : la song phang lien tiep ma ham song co dang sin hoac cos.

$$a(\vec{r},t) = A\cos(\omega t - \vec{k}\vec{r} - \Phi)$$

- So song $\mathrm{k} = \dfrac{2\pi}{\lambda}$

- Vectô song $\vec{k} = k\vec{u}$

Song ñien tö phang ñieu hoa lien tiep la nghiem cua phöông trình Maxwell ma 6 thanh phan cua tröông ñien tö co cung tan so goc ω va cung vectô song $\vec{k}$

Co the bieu dien tröông ñien tö döôi dang phöc :

$$\underline{\vec{E}} = \underline{\vec{E}_0} e^{j(\omega t - \vec{k}\vec{r})}; \qquad \underline{\vec{B}} = \underline{\vec{B}_0} e^{j(\omega t - \vec{k}\vec{r})} \qquad (j^2 = -1)$$

Cac toan tö ñao ham tac dung len tröông phöc töông ñöông vôi phep nhan :

$$\frac{\partial}{\partial t} = j\omega; \qquad \vec{\nabla} = j\vec{k}$$

2. Cau truc cua OPPH trong chan khong :

Bieu dien pt Maxwell bang cach sö dung toan tö nabla $\vec{\nabla}$

$$\vec{\nabla}\vec{E} = 0 \qquad \vec{\nabla}\ \vec{E} = -\partial\ \vec{B}\ /\ \partial T$$

$$\vec{\nabla}\vec{B} = 0 \qquad \vec{\nabla} \wedge \vec{B} = \varepsilon_0 \mu_0 \partial \vec{E}\ /\ \partial t$$

Viet döôi dang phöc :

$$-j\vec{k}.\vec{E} = 0 \quad (1) \qquad -j\vec{k} \wedge \vec{E} = -j\omega\underline{\vec{B}} \quad (3)$$

$$-j\vec{k}.\vec{B} = 0 \quad (2) \qquad -j\vec{k} \wedge \vec{B} = \varepsilon_0\mu_0 j\omega\underline{\vec{E}} \quad (4)$$

Tö phöông trình (1) ta co $\vec{k}.\underline{\vec{E}} = 0 \Rightarrow \vec{u}.\underline{\vec{E}} = 0$

$$\rightarrow \mathrm{Re}(\vec{u}.\underline{\vec{E}}) = 0 \Rightarrow \vec{u}.\mathrm{Re}(\underline{\vec{E}}) = 0$$

$$\Rightarrow \vec{u}.\vec{E} = 0$$

Mot cach töông tö ta cung co $\vec{u}.\vec{B} = 0$

→ Song ñien tö phang ñeu hoa lien tiep trong chan khong la song ngang.

$$(3) \Rightarrow \underline{\vec{B}} = \vec{k} \wedge \underline{\vec{E}} = k.\vec{u} \wedge \underline{\vec{E}} \qquad (3')$$

$$(4) \Rightarrow k\vec{u} \wedge \underline{\hat{B}} = -\frac{\omega}{C^2}\underline{\vec{E}} \qquad (4')$$

Theá (3') vaøo (4') :

$$k.\vec{u} \wedge (\frac{k}{\omega}\vec{u} \wedge \underline{\vec{E}}) = -\frac{\omega}{C^2}\underline{\vec{E}}$$

Maø ta coù : $.\vec{u} \wedge (\vec{u} \wedge \underline{\vec{E}}) = (\vec{u}.\underline{\vec{E}}).\vec{u} - (\vec{u}.\vec{u}).\underline{\vec{E}}) = -\underline{\vec{E}}$

=> $k^2 = \frac{\omega^2}{c^2}$

(3') => $\underline{\vec{B}} = \frac{k}{\omega}\vec{u} \wedge \underline{\vec{E}} = \frac{\vec{u} \wedge \underline{\vec{E}}}{c}$

Laáy phaàn thöïc :

$$\vec{B} = \mathrm{Re}(\frac{\vec{u} \wedge \underline{\vec{E}}}{c}) = \frac{\vec{u} \wedge \vec{E}}{c}$$

=> $(\vec{u}, \vec{E}, \vec{B})$ taïo thaønh moät tam dieän thuaän

Maët khaùc tyû soá giöõa tröôøng ñieän vaø töø laø:

$$\frac{E(M,t)}{B(M,t)} = c$$

Ñieän tröôøng vaø töø tröôøng cuûa OPPH ñoàng pha. Caùc tính chaát treân cuõng ñuùng vôùi OPP.

3. Söï phaân cöïc cuûa OPPH :

Trong OPPH phöông cuûa ñieän tröôøng $\vec{E}$ trong maët phaúng vuoâng goùc vôùi phöông truyeàn soùng $\vec{u}$ chöùa ñöôïc xaùc ñònh. Phöông cuûa vectô $\vec{E}$ ñöôïc goïi laø phöông phaân cöïc cuûa soùng

Xeùt trong heä toaï ñoä Descartes, giaû söû soùng truyeàn theo phöông z :

$$\vec{E} = \begin{cases} E_{0x}\cos(\omega t - kz + \phi_x) \\ E_{0y}\cos(\omega t - kz + \phi_y) \\ 0 \end{cases}$$

Moät khi bieát ñöôïc ñieän tröôøng $\vec{E}$ ta coù theå xaùc ñònh ñöôïc töø tröôøng nhôø caáu truùc cuûa OPPH.

Taïi moät vò trí z = z_0 coá ñònh, ta coù theå vieát söï bieán thieân cuûa ñieän tröôøng nhö sau :

$$E_x = E_{0x}\cos(\omega t)$$

$$E_y = E_{0y}\cos(\omega t - \varphi)$$

Vôùi $\varphi = \varphi_x - \varphi_y$: ñoä treã pha cuûa E_y ñoái vôùi E_x

Ñaàu muùt cuûa vectô ñieän tröôøng dòch chuyeån trong maët phaúng (xOy), treân ñöôøng ellipse coù phöông trình :

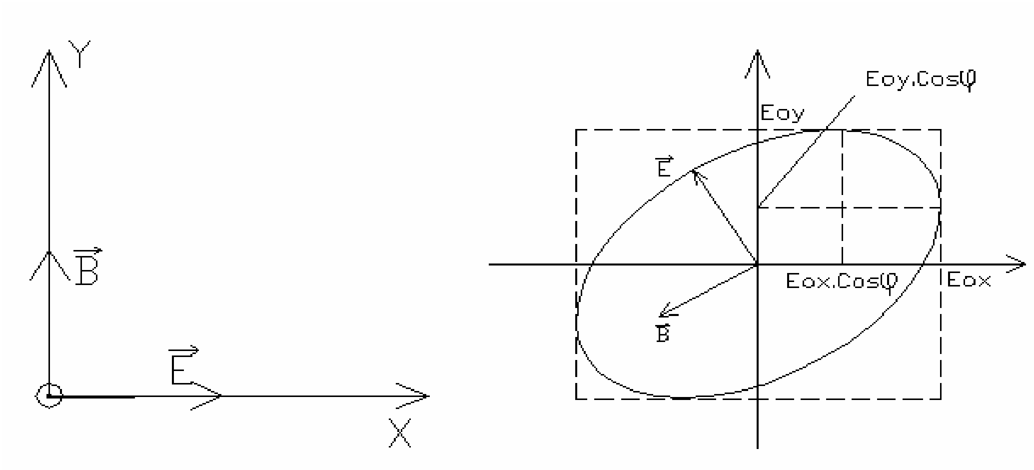
$$(\frac{E_x}{E_{0x}})^2 + (\frac{E_y}{E_{0y}})^2 - 2(\frac{E_x}{E_{0x}})(\frac{E_y}{E_{0y}})\cos\varphi = \sin^2\varphi$$

noi tiep trong hình chöi nhat coi canh $2E_{0x}$ vai $2E_{0y}$

Ñei xac ñònh chieu chuyein ñoing doic theo ellipse, ta xeit vao thôii ñieim t = 0, khi ñoi $E_x = E_{0x}$ vai :

$$\left(\frac{dE_y}{dt}\right)_{t=0} = E_{0y}\omega.\sin\varphi$$

$\Rightarrow$ chieu quay ñöôic chæ ra bôii dau cua sinφ .



- Neu chieu quay thuan chieu kim ñoing hoi: soing phan cöïc ellipse trai, sinφ > 0
- Neu chieu quay ngöôïc chieu kim ñoing hoi: soing phan cöïc ellipse phai, sinφ < 0
- Neu φ = 0 hoac φ = $\pm\pi$,ñau mut cua $\vec{E}$ dòch chuyein trein ñöôøng thaing xac ñònh, ta coi phan cöïc thaing

Noi chung moit soing phan cöïc ellipse coi thei xem la toing cua 2 soing phan cöïc thaing theo hai phöông vuoing goic vôii nhau => moi soing ñien töi trong chan khoing la söi toing hôïp cua cac soing phaing ñieu hoa lien tiep phan cöïc thaing.

- Neu φ = $\pm\pi$ /2 vai $E_{0x} = E_{0y}$ ta coi phan cöïc troøn.

4. Söi truyein naing löôïng cua OPPH:

a) Mat ñoi naing löôïng cua tröôøng ñien töi :

$$e = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2.\mu_0}$$

Ñoi vôii OPPH : $B = \frac{E}{c}$ => $e = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$

⇨ naing löôïng ñöôïc phan boi ñeu döôii daing ñiein vai töi.

Ñoi vôii moit soing OPPH truyein theo phöông cua truic Ox ,tröôøng ñiein töi coi daing:

$$\underline{\vec{E}} = \underline{\vec{E}_0} e^{j(\omega t - kx)}$$

$$\underline{\vec{B}} = \frac{\vec{u}_x \wedge \underline{\vec{E}_0}}{c} e^{j(\omega t - kx)}$$

Giaù trò trung bình cuûa e:

$$< e > = < \varepsilon_0 E^2 > = \varepsilon_0 \frac{1}{2} \mathrm{Re}\left(\underline{\vec{E}}.\underline{\vec{E}^*}\right) = \frac{\varepsilon_0}{2} \left|\vec{E}_0\right|^2$$

b) Vectô Poynting:

Coâng suaát cuûa soùng ñieän töø ñi qua moät ñôn vò dieän tích baèng doøng cuûa vectô Poynting:

$$\vec{\Pi} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}$$

(doøng naêng löôïng ñi qua dieän tích S : $\Phi = \int_S \vec{\Pi}.d\vec{S}$)

Ñoái vôùi OPP $\vec{\Pi} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{\vec{E} \wedge (\vec{u} \wedge \vec{E})}{\mu_0 c} = c\varepsilon_0 E^2 \vec{u}.$

Ñoái vôùi soùng OPPH coù taàn soá ω, giaù trò trung bình $<\Phi>$ cuûa coâng suaát truyeàn qua maët S :

$$<\Phi> = <\vec{\Pi}>.\vec{S} = 1/2\mathrm{Re}\left(\frac{\underline{\vec{E}} \wedge \underline{\vec{B}^*}}{\mu_0}\right).\vec{S} = \frac{1}{2} c\varepsilon_0 \left|\vec{E}_0\right|^2 .S$$

Ghi chuù: $<\vec{E}(t) \wedge \vec{B}(t)> = \frac{1}{2}\mathrm{Re}(\underline{\vec{E}}(t) \wedge \underline{\vec{B}}^*(t)) = \frac{1}{2}\mathrm{Re}(\underline{\vec{E}}_m(t) \wedge \underline{\vec{B}}^*{}_m(t))$

$$= \frac{1}{2}\mathrm{Re}(\vec{E}_m \wedge \vec{B}_m e^{j(\varphi_1 - \varphi_2)}) = \frac{1}{2} \vec{E}_m \wedge \vec{B}_m \cos(\varphi_1 - \varphi_2)$$

c) Vaän toác truyeàn naêng löôïng:

Goïi v_e laø vaän toác truyeàn naêng löôïng.

$<\Pi>.S.\delta t$: naêng löôïng truyeàn qua dieän tích S vuoâng goùc vôùi phöông truyeàn soùng trong khoaûng thôøi gian δt

$S.v_e.\delta t < e >$: naêng löôïng chöùa trong theå tích $S.v_e.\delta t$

$$S.v_e.\delta t < e > = < \Pi > .S.\delta t$$

=> $v_e = \frac{< \Pi >}{< e >}$

Xeùt tröôøng hôïp cuûa OPPH : $v_e = c$

$$\vec{E} = E_{0y} \cos(\omega t - kx)\vec{u}_y \pm E_{0z} \sin(\omega t - kx)\vec{u}_z$$

=> $\vec{\Pi} = \frac{< \vec{E}^2 >}{\mu_0 c}\vec{u} = \frac{\vec{u}}{\mu_0 c} < E_{0y}^2 \cos^2(\omega t - kx) + E_{0z}^2 \sin(\omega t - kx) > = \left(\frac{E_{oy}^2 + E_{0z}^2}{2\mu_0 c}\right).\vec{u}$

$$< e > = \varepsilon_0 < E^2 > = \varepsilon_0 < E_{0y}^2 \cos^2(\omega t - kx) + E_{0z}^2 \sin(\omega t - kx) > = \varepsilon_0 \frac{E_{0y}^2 + E_{0z}^2}{2}$$

- Vectô Poynting phöc : $\vec{\Pi} = \dfrac{\underline{\vec{E}} \wedge \underline{\vec{B}}^*}{\mu_0}$

$$\underline{\vec{E}} = \underline{\vec{E}}_0.\exp\left[j\left(\omega t - \vec{k}\vec{r}\right)\right]$$

Ñoi vôi OPPH :

$$\underline{\vec{B}} = \frac{\vec{u} \wedge \underline{\vec{E}}_0}{c}.\exp\left[j\left(\omega t - \vec{k}\vec{r}\right)\right]$$

$$\underline{\vec{B}}^* = \frac{\vec{u} \wedge \underline{\vec{E}}_0^*}{c}.\exp\left[-j\left(\omega t - \vec{k}\vec{r}\right)\right]$$

=> $\underline{\vec{\Pi}} = \dfrac{\underline{\vec{E}} \wedge \underline{\vec{B}}^*}{\mu_0} = \underline{\vec{E}}_0 \wedge (\dfrac{\vec{u} \wedge \underline{\vec{E}}_0^*}{\mu_0 c}) = (\dfrac{\underline{E}_0.\underline{E}_0^*}{\mu_0 c}).\vec{u} = \dfrac{E_0^2}{\mu_0 c}.\vec{u}$

=> $< d\phi > = < \vec{\Pi} > d\vec{S}$

Vôi $< \vec{\Pi} > = \dfrac{< E^2 >}{\mu_0 c}\vec{u} = \dfrac{c\varepsilon_0 E_0^2}{2}\vec{u}$